On the Emergence of Relational Phase in Collapse-Selection Dynamics

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Abstract

We provide a minimal structural account of the emergence of relational phase within collapse-selection dynamics. Rather than assuming complex amplitudes, we show that a phase-like quantity arises naturally as a consistency condition for composing admissible transition chains under finite invariance. This establishes phase accumulation as a structural necessity for coherent composition across admissible relational sectors.

1 Motivation

In collapse-selection formulations, transition weights are typically expressed in the form:

$$T_{\alpha\beta} = A_{\alpha\beta} e^{-\Gamma_{\alpha\beta}} e^{i\Theta_{\alpha\beta}}. \tag{1}$$

While accessibility $A_{\alpha\beta}$ and collapse-loss $\Gamma_{\alpha\beta}$ admit direct structural interpretation, the phase term $\Theta_{\alpha\beta}$ is often introduced by analogy with quantum mechanics.

The purpose of this note is to show that a phase-like structure arises naturally from the requirements of composability, invariance, and distinguishability in admissible transition chains.

2 Admissible Transition Composition

Let $C_\alpha, C_\beta, C_\gamma$ be admissible configuration sectors.

Consider two composable transitions:

$$C_\gamma \to C_\beta \to C_\alpha. \tag{2}$$

The corresponding weights must satisfy a composition rule:

$$T_{\alpha\gamma} \sim T_{\alpha\beta} T_{\beta\gamma}, \tag{3}$$

up to normalization and coarse-graining.

If transition weights were purely real and positive:

$$T_{\alpha\beta} = A_{\alpha\beta} e^{-\Gamma_{\alpha\beta}}, \tag{4}$$

then composition would be strictly multiplicative and path-independent.

However, this leads to an ambiguity:

> Different admissible paths between the same sectors would be indistinguishable under projection.

This violates finite invariance, since distinct relational histories would collapse to identical descriptive structure.

3 Resolution via Phase Structure

To preserve distinguishability of admissible paths, transition weights must carry additional relational structure.

Introduce a phase function $\Theta_{\alpha\beta}$ such that:

$$T_{\alpha\beta} = A_{\alpha\beta} e^{-\Gamma_{\alpha\beta}} e^{i\Theta_{\alpha\beta}}. \tag{5}$$

We impose the composition constraint:

$$\Theta_{\alpha\gamma} \sim \Theta_{\alpha\beta} + \Theta_{\beta\gamma}. \tag{6}$$

Thus, phase acts as an additive relational quantity tracking accumulated structural differences across admissible chains.

4 Interference as Consistency Condition

Consider two admissible chains from C_i to C_f:

$$\gamma_1 : C_i \to C_1 \to C_f, \qquad \gamma_2 : C_i \to C_2 \to C_f. \tag{7}$$

The total transition weight is:

$$K = T_1 + T_2, \tag{8}$$

with

$$T_j = A_j e^{-\Gamma_j} e^{i\Theta_j}. \tag{9}$$

The observable probability becomes:

$$|K|^2 = |T_1|^2 + |T_2|^2 + 2\mathrm{Re}(T_1 T_2^*). \tag{10}$$

Thus, interference arises as a direct consequence of phase compatibility between admissible relational paths.

Without phase structure, distinct admissible paths would collapse into indistinguishable contributions, eliminating interference.

5 Interpretation

Phase is not an independent dynamical variable.

It is:

> A relational bookkeeping structure required to preserve consistency of admissible transition composition under projection.

More specifically:

- Accessibility encodes whether transitions can occur,
- Collapse-loss encodes stability under constraint,
- Phase encodes relational distinction between admissible paths.

Thus, phase arises from the need to maintain distinguishability of relational structure under finite-resolution projection.

6 Summary

We have shown that:

- phase is required to preserve distinguishability of admissible transition chains,
- phase composition follows from consistency under chain concatenation,
- interference emerges as a structural consequence of phase compatibility,
- complex-valued transition weights are therefore not arbitrary, but minimally required.

  Relational phase is the minimal structure required to preserve path distinguishability under collapse-selection and projection.